Fundamentals of Solid State Physics

Electron Motion in a Magnetic Field

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Outline

- **Cyclotron resonance 回旋共振**
  - **measure the effective mass m^***

- **Hall effect 霍尔效应**
  - **measure the carrier density n**

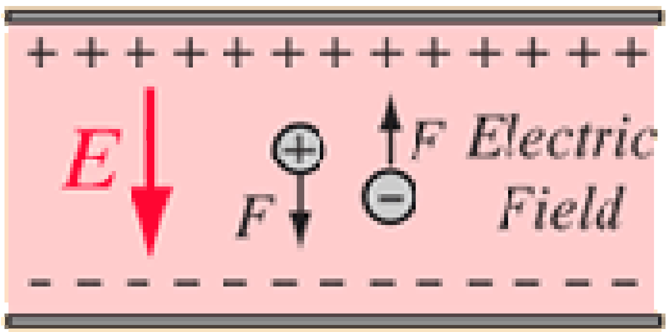
Electrons in Electromagnetic Fields

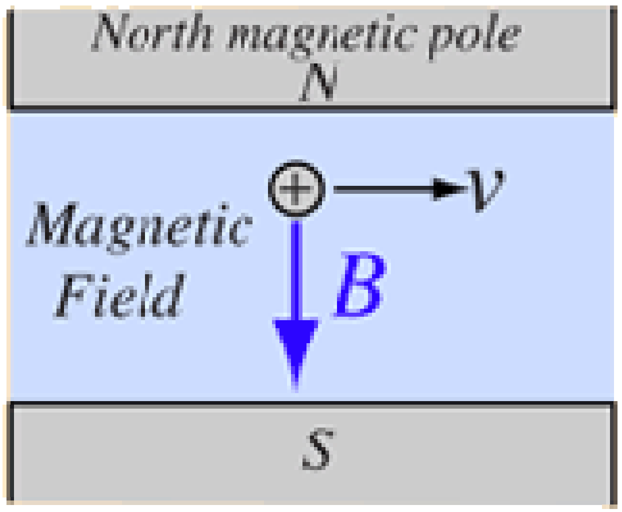
- **Lorentz force**

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

electric force

magnetic force





Cyclotron resonance 回旋共振

Lorentz force

$$\mathbf{F} = -e\mathbf{E} - e\mathbf{v} \times \mathbf{B}$$

→

When E = 0, B = B_z

$$\mathbf{F} = m^* \frac{d\mathbf{v}}{dt} = -e\mathbf{v} \times \mathbf{B}$$

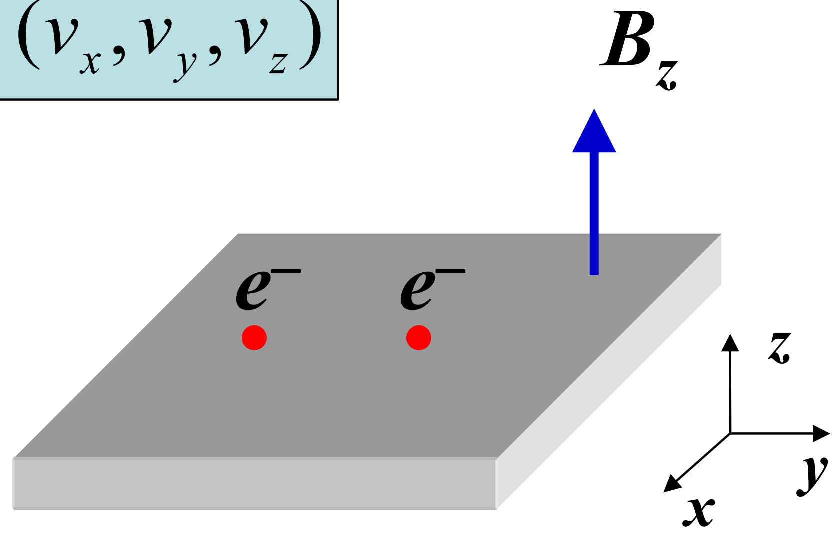
$$\mathbf{v} = (v_x, v_y, v_z)$$

→

$$\frac{dv_x}{dt} = -\frac{eB_z}{m^*} v_y$$

$$\frac{dv_y}{dt} = \frac{eB_z}{m^*} v_x$$

$$v_z = 0$$



define

$$\omega_c = \frac{eB_z}{m^*}$$

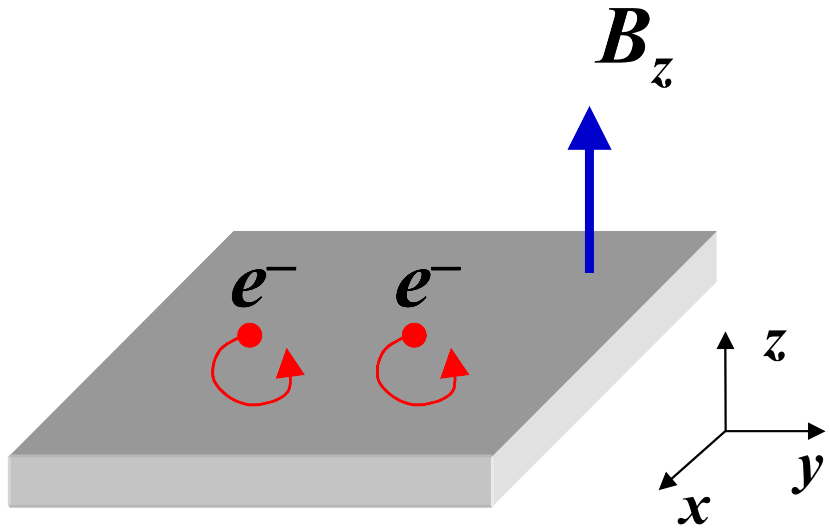
Cyclotron resonance 回旋共振

$$v_x = v_0 \cos(\omega_c t)$$
$$v_y = v_0 \sin(\omega_c t)$$
$$v_z = 0$$

cyclotron frequency

$$\omega_c = \frac{eB_z}{m^*}$$

electrons move in a circle



B **- magnetic field (T)**
m* **- effective mass (kg)**
ω **- angular frequency (rad/s)**
ν **- frequency (Hz)**
$\omega = 2\pi\nu$
e = 1.6*10^{-19} C

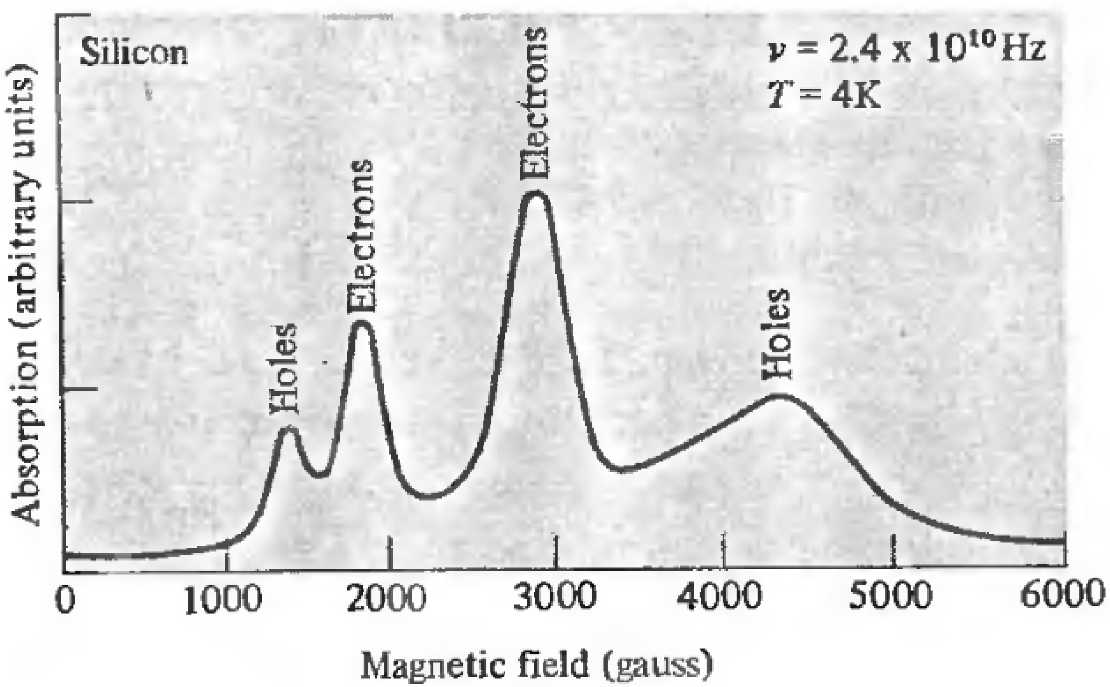
Cyclotron resonance 回旋共振

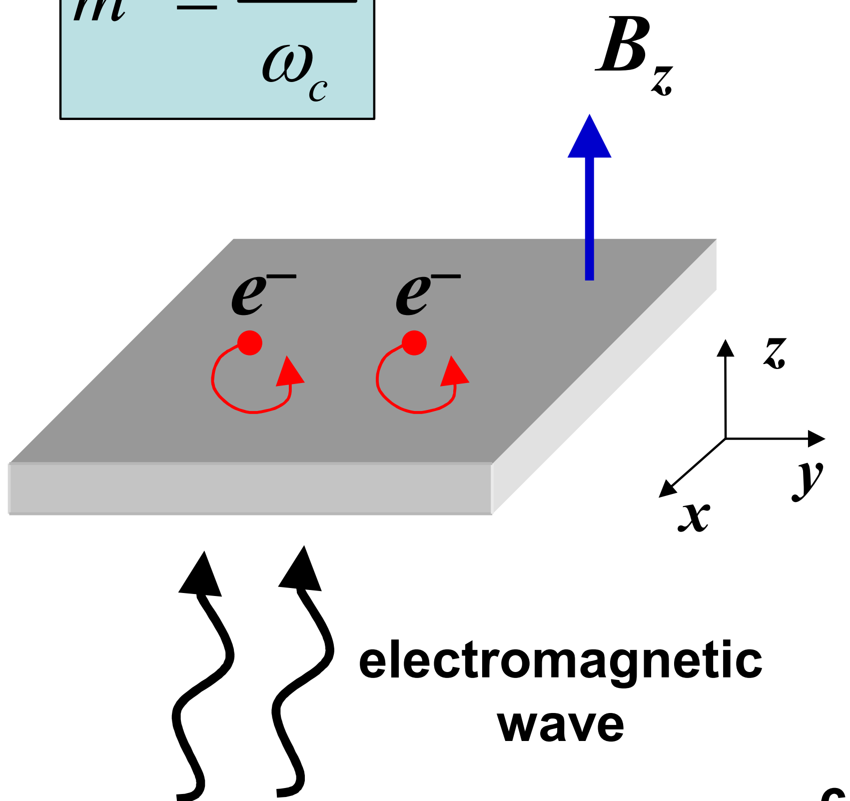
absorption peaks of EM wave at cyclotron frequency

$$\omega_c = \frac{eB_z}{m^*}$$

so we can measure effective mass

$$m^* = \frac{eB_z}{\omega_c}$$





Ashcroft & Mermin, p572

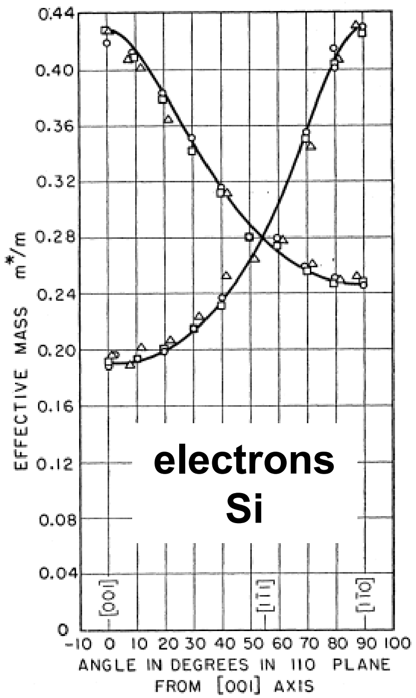
Cyclotron resonance 回旋共振

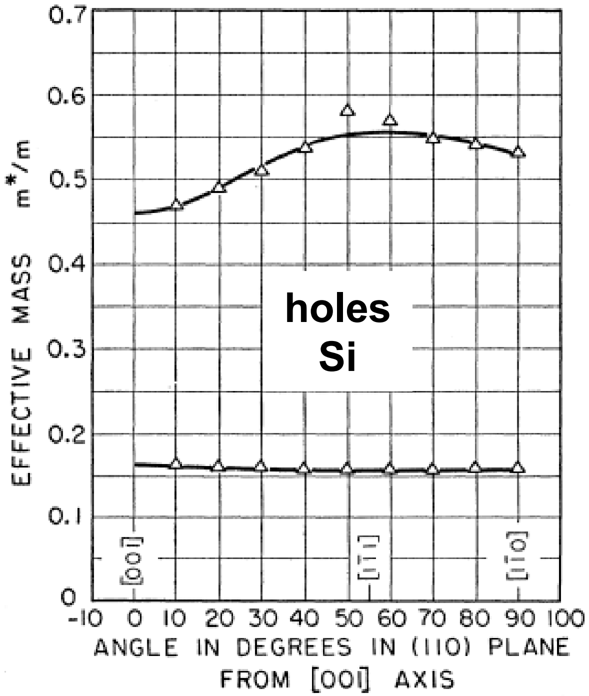
absorption peaks of EM wave at cyclotron frequency

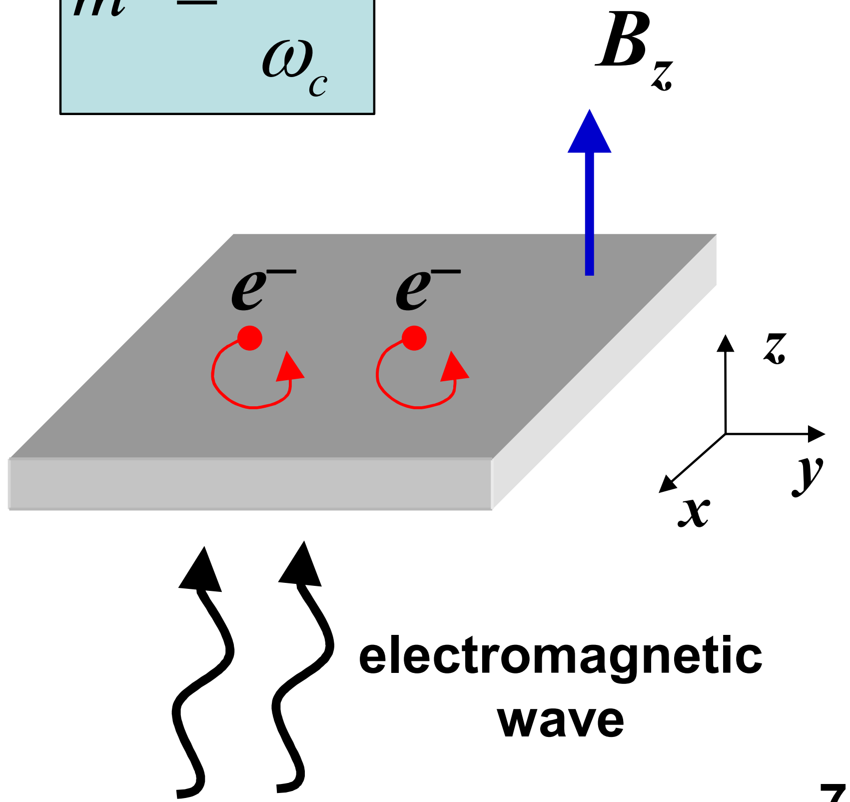
$$\omega_c = \frac{eB_z}{m^*}$$

so we can measure effective mass

$$m^* = \frac{eB_z}{\omega_c}$$



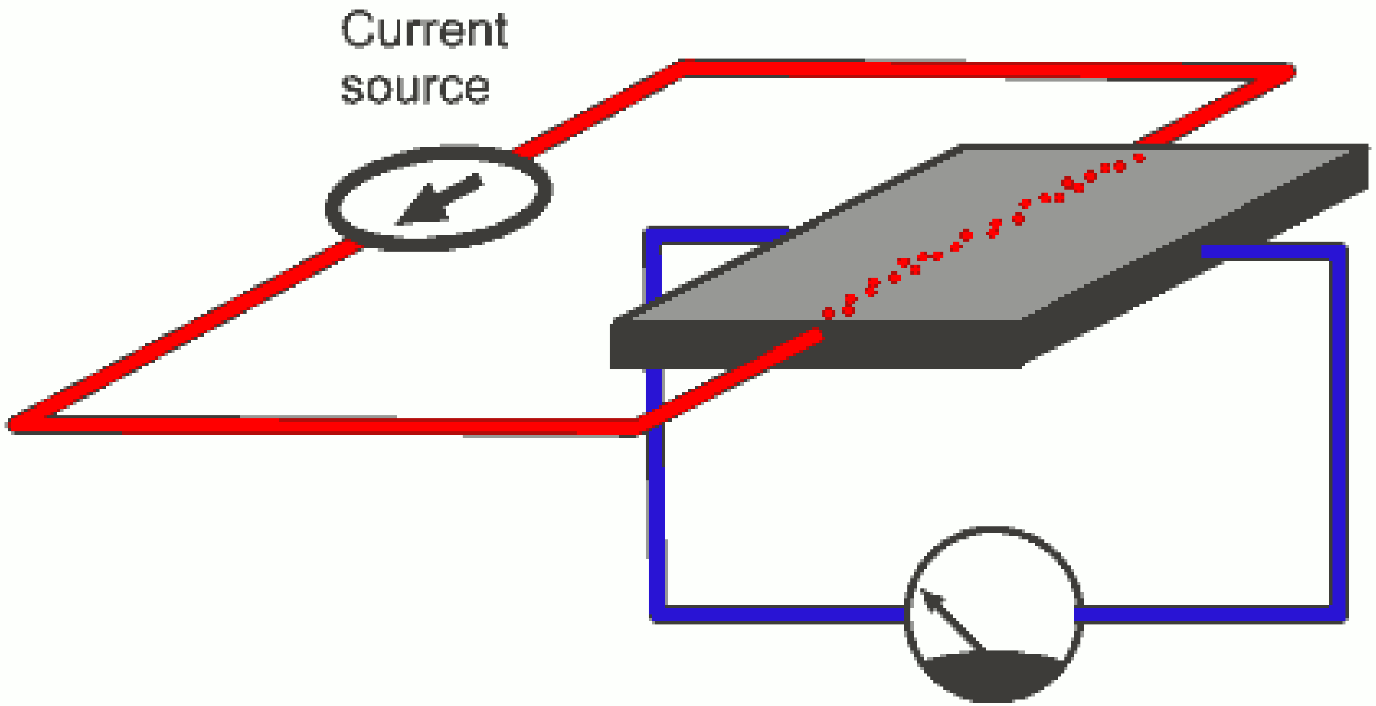




G. Dresselhaus, et. al., *Phys. Rev.* **98**, 368 (1955)

Hall Effect 霍尔效应

- A current flows through a conductor
- V_H is generated when applying B_z



V_H - Hall voltage

Hall Effect 霍尔效应

- **A current flows through a conductor**
- **V_H is generated when applying B_z**

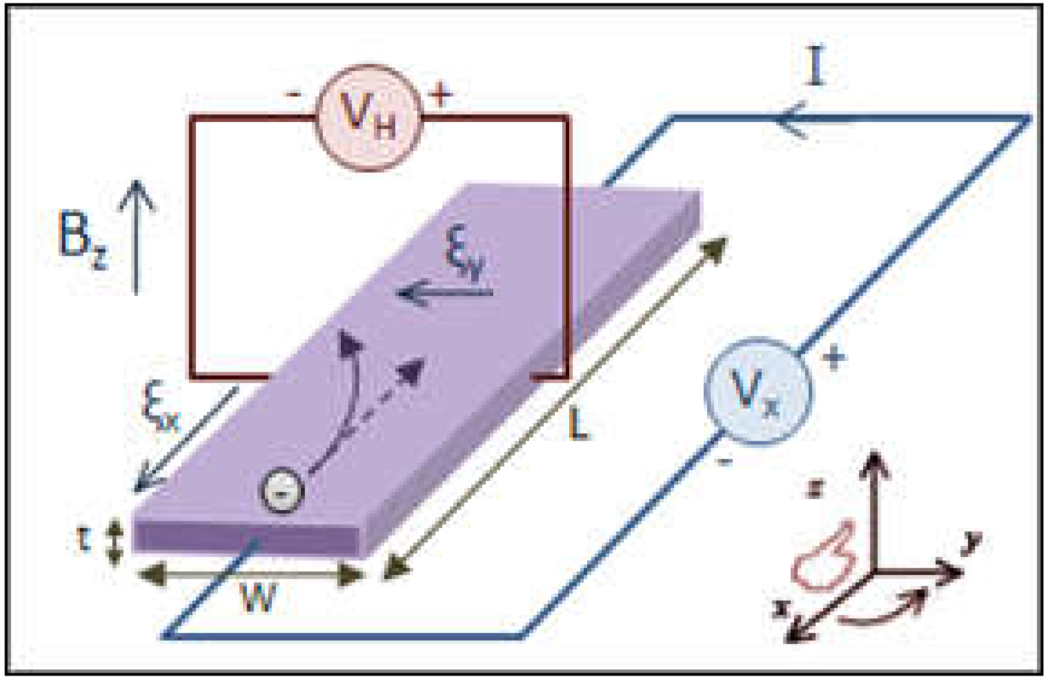
Lorentz Force at equilibrium

$$eE_y = ev_x B_z$$

$$j_x = nev_x$$

$$E_y = \frac{1}{ne} j_x B_z = R_H j_x B_z$$

R_H - Hall coefficient



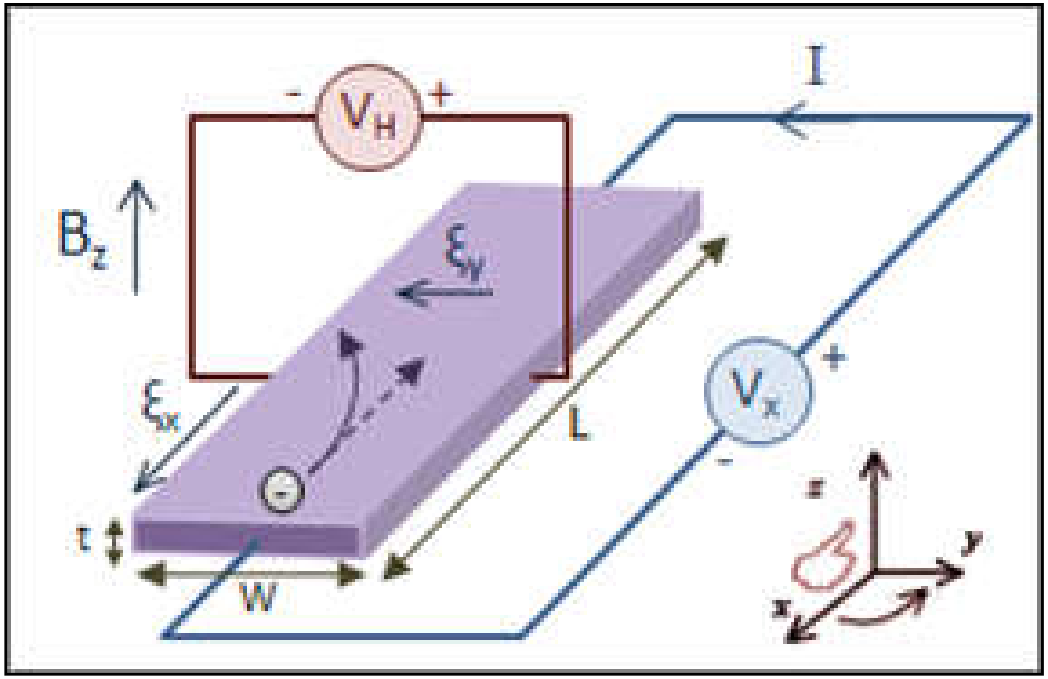
By definition:
positive charge: $R_H > 0$
negative charge: $R_H < 0$

https://www.tf.uni-kiel.de/matwis/amat/mw2_ge/kap_2/backbone/r2_1_3.html

Hall Effect 霍尔效应

- **A current flows through a conductor**
- **V_H is generated when applying B_z**

$$E_y = \frac{1}{ne} j_x B_z = R_H j_x B_z$$



p-doping | **n-doping**

$$R_H = \frac{1}{p_v e}$$

$$R_H = -\frac{1}{n_c e}$$

positive | **negative**

R_H only depends on the carrier density

Hall Effect 霍尔效应

- R_H **only depends on the carrier density**
  - **semiconductors have much stronger Hall effects than metals**

$$n(\text{metal}) \gg n(\text{semiconductor})$$

$$R_H(\text{metal}) \ll R_H(\text{semiconductor})$$

Hall Effect 霍尔效应

- Examples

Metals	R_H (unit: 1/*ne*)
Li	–0.8
Na	–1.2
K	–1.1
Cu	–1.5
Ag	–1.3
Au	–1.5
Mg	+0.4
Al	+0.3

n - density of valance electrons

p-doping

$$R_H = \frac{1}{p_v e}$$

positive

n-doping

$$R_H = -\frac{1}{n_c e}$$

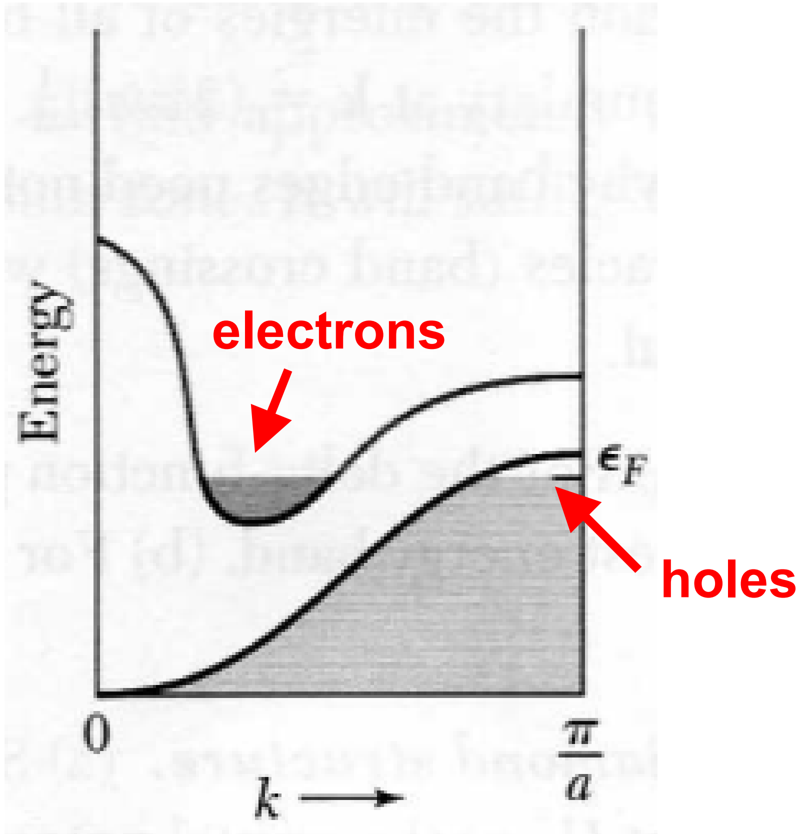
negative

Silicon	R_H (unit: m^3/C)
Boron doped $N_A = 10^{16}$ cm^{-3}	6×10^{-4}
Arsenic doped $N_D = 10^{16}$ cm^{-3}	-6×10^{-4}

Hall Effect 霍尔效应

- **Mg and Al have positive Hall coefficients**
  - **conduction is partially contributed by holes**

Metals	R_H (unit: 1/*ne*)
Li	–0.8
Na	–1.2
K	–1.1
Cu	–1.5
Ag	–1.3
Au	–1.5
Mg	+0.4
Al	+0.3



n **- density of valance electrons**

Summary

- **Cyclotron resonance 回旋共振**
  - **measure the effective mass *m****

$$m^* = \frac{eB_z}{\omega_c}$$

- **Hall effect 霍尔效应**
  - **measure the carrier density *n***

$$R_H = \frac{1}{p_v e}$$

$$R_H = -\frac{1}{n_c e}$$

Thank you for your attention